UNDERSTANDING THE EX-ANTE COST OF LIQUIDITY IN THE LIMIT ORDER BOOK: A NOTE*

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This paper estimates a new measure of liquidity costs in a market driven by orders. It represents the cost of simultaneously buying and selling a given amount of shares, and it is given by a single measure of ex-ante liquidity that aggregates all available information in the limit order book for a given number of shares. The cost of liquidity is an increasing function relating bid-ask spreads with the amounts available for trading. This measure completely characterizes the cost of liquidity of any given asset. It does not suffer from the usual ambiguities related to either the bid-ask spread or depth when they are considered separately. On the contrary, with a single measure, we are able to capture all dimensions of liquidity costs on ex-ante basis.

Key words: liquidity function, liquidity cost, open limit order book, bid-ask spread, depth.

JEL classification: G14.

It is clearly recognized that liquidity benefits the individual investors in securities markets. Generally speaking we all understand that liquidity somehow reflects the ability to trade basically costlessly. Hence liquid markets should be able to accommodate large amounts of trading without distorting impacts on prices.

Unfortunately, however, what this really means in practice, or even in a formal analytical sense is much less clear. We might use the idea of Kyle (1985) in

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which liquidity measures the order flow needed to change prices one unit. Note
that this concept lies on the use of the aggregated order flow, so that it might be
argued that from the point of view of individual investors, the relative bid-ask
spread is a more appropriate measure of liquidity. Of course, the market will be
more liquid the lower the bid-ask spread.

Nevertheless, Easley and O’Hara (1987) and O’Hara (1995) argue that there
may not be a single spread as long as prices vary with trade size. In fact, they
show that the bid-ask spread for large trades may be considerably larger than the
spread for small trades. This is a consequence of the spread arising as a compen-
sation for the risk of trading with individuals who have superior information. The
spread needs not to be constant across quantities.

This is the basis of the reasoning provided by Lee, Mucklow and Ready
(1993) who argue convincingly in favor of the bidimensionality aspect of liquidi-
ty. The cost faced by an individual who wishes to trade simultaneously buying
and selling shares, which of course reflects the cost of immediacy, must have a
quantity dimension, given that this cost depends on the size of the operation.
Therefore, liquidity effects are unambiguous only when we observe a spread in-
crease (decrease) and a simultaneous depth decrease (increase), where depth is
the number of shares available at each side of the market.

Somehow surprisingly, most papers analyzing the behavior of liquidity
throughout the day or week have just studied the relative quoted spread or the rel-
ative effective spread, without considering the effects of trade size. Important ex-
teptions are Lee, Mucklow and Ready (1993), Glosten (1994), Lin, Sanger and
Booth (1995), Blanco (1999), Pascual, Escribano and Tapia (2004), Goldstein and
Kavajecz (2000), and Benston, Irvine and Kandel (2000). In any case, except the
work by Blanco (1999) who employs data from the Spanish Stock Exchange, and
Benston, Irvine and Kandel (2000) who use the limit order book from the Toronto
Stock Exchange, these papers are based on NYSE firms in which the role of the
specialist proving liquidity is a key characteristic of the market.

This paper proposes and estimates a single measure of ex-ante liquidity that
is an increasing function relating bid-ask spreads with the amounts available for
trading at each side of the market. As an ex-ante measure, it represents the cost of
simultaneously buying and selling a given amount of shares at which a transaction
could be immediately carried out. It may either facilitates trading precisely be-
cause ex-ante liquidity is high, or it will discourage transactions when is low. This
function will be called the liquidity function, and it is developed in the context of
a continuous auction system driven by orders. In other words, we exploit the op-
portunities provided by a continuous stock exchange market in which liquidity is
provided by orders observed in an electronic open limit order book. These order
driven trading systems are employed in the Tokyo Stock Exchange, the Paris
Bourse, the Toronto Stock Exchange, and the Spanish Stock Market among oth-
ers. Our estimation aggregates all information available in the limit order book
into a single liquidity cost for a given number of shares. It simultaneously takes
into account the price and the quantity dimension of liquidity, so that it provides
investors with a measure that allows comparisons between markets and stocks.
This is certainly important and interesting, and it may have serious implications
for estimating the liquidity premium in the context of asset pricing models.
The markets driven by orders with an electronic open limit order book, provided that data are available, are particularly well suited to study liquidity by considering both the price and the quantity dimensions as it should be done. In particular, we have the complete limit order book for July 1999 for five stocks of different market value trading in the Spanish continuous auction exchange. As in the paper by Biais, Hillion and Spatt (1995), and De Jong, Nijman and Röell (1996) for the Paris Bourse, we have the whole record of the limit order quantity at the five best prices on both the bid and the ask side of the market. However, the objective of their papers is very different from ours. We estimate the liquidity function and, to illustrate, we briefly discuss the cross-sectional differences among five stocks with five alternative levels of market capitalization. Therefore, the availability of the five levels of prices and their corresponding quantities of the order book allows us to fully understand and describe liquidity as a function of the number of shares. The note is devoted mainly to discuss and estimate the liquidity function. The seasonal behavior of the liquidity cost for a given number of shares, its determinants, further statistical analysis, and other properties as its potential to predict the future order flow remains for future research.

It should be clear that we do neither employ transaction prices or actual trade sizes to analyze our measure of liquidity costs. Rather, the impact of private information attributable to adverse selection, as well as the competing different opinions among liquidity providers, originates the innovations occurred in the limit order book. We just summarize the consequences of these entries into a single measure of liquidity cost for a given number of shares.

Moreover, it is interesting to point out that, as expected, the relative bid-ask spread systematically overestimates the true cost of liquidity given by our measure, which is of course increasing in size. However, the cross sectional classification of liquidity with both two measures is practically the same. This is a useful result because it somehow justifies the traditional literature connecting microstructure and asset pricing.

Next section describes the data employed in our analysis; the liquidity function is presented in Section 3 of the paper, while the empirical illustrations are reported in Section 4. Finally, we summarize our results and provide some conclusions in Section 5.

1. DATA

The open limit order book contains information about the five best levels of prices of selling and buying orders over all assets in each instant. For each of these levels and for each market side we have information on the best price, volume of shares outstanding (depth) and number of orders which supports such volume. When a modification on any of these variables occurs, the limit order book shows us the new values of the variables, while the time stamps indicates exactly the time of this change (approximated by tenths of a second).

In order to consider a wide range of the market, we analyze the behavior of five alternative stocks chosen according to its market capitalization. These assets are all included in the Spanish IBEX-35 market index. The IBEX-35 is a value-
weighted index comprising the 35 most frequently traded Spanish stocks of the continuous market. At the end of June 1999, all stocks comprising the index were classified into five portfolios according to its market value. The largest stocks in each of the five size-sorted portfolios were finally chosen. Following this criterion, TELEFÓNICA [TEF] (which represents 20,46% of the IBEX-35 index), GAS NATURAL [CTG] (4,32), ACESA [ACE] (1,24), ACERINOX [ACX] (0,68) and TELEPIZZA [TPZ] (0,44) were selected. Our sample period covers all trading days of July 1999.

The database has been checked looking for errors. Specifically, some observations where the bid was greater than the ask price has been removed from the sample (these errors were always found at the very beginning of the day). Thus for each modification in the order book we have 30 variables, six for each level (three on the buy side and three on the sell side) and the time of the change. The final number of observations (changes in the limit order book) during July 1999 was for each stock as follows: TELEFÓNICA, 86937; GAS NATURAL, 22823; ACESA, 12109; ACERINOX, 9603; and TELEPIZZA, 30471.

2. THE LIQUIDITY FUNCTION

As pointed out by De Jong, Nijman and Röell (1996), the trading mechanism operating in markets driven by orders can be formally described by the ideal electronic open limit order book framework proposed by Glosten (1994). This author presents a theoretical model of price revisions due to the information conveyed by trading throughout the limit order book mechanism. This is the framework in which our liquidity cost function is estimated. Glosten develops both average and marginal price functions from the point of view of the agent providing liquidity. Alternatively, we may understand these functions as revenue functions. Blanco (1999) discusses similar functions, and shows that what he calls the supply function (equivalent to Glosten’s average revenue function) is constant (and equal to the ask price, or first level of prices) for the number of shares less or equal to the volume of the first level, and increasing and concave for a greater number of shares, with a different concavity for the volume corresponding to the remainder levels. On the other hand, the demand function is first constant, and then decreasing and convex, with also a different convexity for the volume associated with other levels.

We also employ the above framework by noting that our liquidity function can be derived using the bid and ask prices available at each of the five levels of the limit order book, and their corresponding volumes or depth.

Contrary to the perspective adopted by either Glosten or Blanco, our supply function is defined from the point of view of the investors willing to buy at the price shown in the limit order book. Hence, we define the average cost function (AC) as the unit cost of buying a determined volume of shares:

\[
AC(n) = \frac{\sum_{i=1}^{n} p_{ask}(i)}{n}
\]  

[1]
where $P_{\text{ask}}(i)$ is the price investors should pay for the $i$-th share, and $n$ is the number of shares they want to buy. Say that there are three levels of ask prices and their corresponding accumulated quantities or total available shares at those prices: $P_{a1} < P_{a2} < P_{a3}$, and the accumulated quantities $Q_{a1} < Q_{a2} < Q_{a3}$. The average cost function would be given by $P_{a1}$ if $n < Q_{a1}$; $[P_{a1} Q_{a1} + P_{a2} (n - Q_{a1})]/ n$ if $Q_{a1} \leq n \leq Q_{a2}$, and finally it would be given by $[P_{a1} Q_{a1} + P_{a2} (Q_{a2} - Q_{a1})] + P_{a3} (n - Q_{a2})/ n$ if $Q_{a2} \leq n \leq Q_{a3}$. It can be shown that this is an increasing and (step-wise) concave function after the shares available at the first level of prices.

From the other side of the market, we define the average revenue function (AR) as the unit revenue of selling a determined volume of shares:

$$AR(n) = \frac{\sum_{i=1}^{n} P_{\text{bid}}(i)}{n}$$

[2]

where $P_{\text{bid}}(i)$ is the price investors would receive for the $i$-th share and $n$ is the number of assets they want to sell. Again, it can be shown that this function is constant (and equal to the first level of prices) for the number of shares less or equal to the volume of the first level, and decreasing and (step-wise) convex for greater number of shares.

We are now in a position of defining the liquidity function as a new measure of liquidity. It measures the ex-ante relative costs of buying and selling simultaneously a given number of shares:

$$L(n) = \frac{AC(n) - AR(n)}{V^*}$$

[3]

where $V^*$ is a proxy for the true value of the asset, introduced uniquely to make liquidity costs comparable across assets with different prices, and is defined as:

$$V^* = \frac{\sum_{j=1}^{5} \{[P_{\text{ask}}(j) \times Q_{\text{ask}}(j)] + [P_{\text{bid}}(j) \times Q_{\text{bid}}(j)]\}}{\sum_{j=1}^{5} [Q_{\text{ask}}(j) + Q_{\text{bid}}(j)]}$$

[4]

where $Q_{\text{ask}}(j)$ is the number of available selling shares at the $j$-th level, while $Q_{\text{bid}}(j)$ is the number of available buying shares at the $j$-th level.

As before, it can be shown that the liquidity function is constant for a volume less or equal as the lowest number of shares at either side of the market, and increasing at larger operating sizes.

Blanco (1999) also proposes a similar measure, where the true value is just the midpoint of the average cost and revenue functions for each number of shares. However, since the spread needs not be symmetric around the true value of the stock, it does not seem to be correct to simply use the midpoint of the spread as
the market price. Moreover, it should be noticed that our definition of the stock’s true value is unique independently of the number of shares, whereas this is not the case in Blanco’s definition. However, although being no essential to the contribution of the paper, some potential shortcomings could arise in using $V^*$ as a proxy for the true value of assets.

Our definition allows us to distinguish each market side. In particular, the relative liquidity cost of buying a given number of shares ($n$) is defined as:

$$LB(n) = \frac{AC(n) - V^*}{V^*}$$  [5]

and the relative liquidity cost of selling a determined number of shares as:

$$LS(n) = \frac{V^* - AR(n)}{V^*}$$  [6]

It should be pointed out that both functions are increasing in the number of shares.

Moreover,

$$L(n) = LB(n) + LS(n)$$  [7]

The liquidity function given by equation [3] completely characterized the ex-ante cost of liquidity of any given asset. It does not suffer from the usual ambiguities related to either the bid-ask spread or depth when they are considered separately. With a single measure, we are able to capture all dimensions associated with liquidity costs. Its estimation, as we explained in the next section of the paper, employs a superior set of information than the more traditional measures of liquidity. All information available in the open limit order book is necessary to calculate equation [3]. This implies that our measure is particularly well suited for markets driven by orders.

Recently, and working independently, Benston, Irvine and Kandel (2000) propose a similar measure of ex-ante liquidity. However, they estimate the cost for trade sizes calculated in dollars instead of using either the number or percentage of shares. Moreover, their suggestion is less intuitive than our measure of liquidity cost since they do not employ the theoretical framework developed by Glosten (1994) for the electronic mechanisms of trading. In any case, their proposal referred as the Cost of a Round Trip trade of D dollars is a useful tool in predicting future order flow.

3. **Empirical results: a descriptive analysis**

In the empirical results we report below, and in order to allow for fair comparisons between different moments of the day and stocks, it was decided to normalize the volume of shares in each liquidity function by the total number of
shares at each side in the limit order book at the five levels. Thus, the horizontal axis of the liquidity functions, we actually estimate, measures the percentage of shares over the total limit order book instead of the number of shares. This implies that we provide the *ex-ante* cost of liquidity in the sense of knowing the cost of buying and selling simultaneously a given percentage of the limit order book.

Such normalization fails to capture the differences in depth between assets, both sides of the book and different moments of time. Of course, this is not the case for the liquidity function, \( L(n) \), introduced in [3] which is unique for each asset and each moment of the day. Alternative normalizations based on asset activity, the median size of daily transactions (Normal Market Size), or previous volume traded, might be analyzed. However, given the probability of finding missing values when depth is low (similar to the case of no normalization), and despite of its shortcomings, this work normalizes by the total number of shares at each side of the book.

It must be noted that higher percentages imply a movement from the first levels of the book to higher levels where the conditions at each side of the market are worst. Of course, as expected, this indicates that the liquidity functions we report are always increasing.

Table 1 contains the average cost of *ex-ante* liquidity for each stock in the sample, and it is obtained taking into account all instants in which there is a new entry in the open limit order book. It should be noted that TEF, the largest stock in the Spanish continuous market, has 86937 entries in the book during the month. However, the number of entries is not directly related to market capitalization. TPZ, the smallest stock in our sample, has 30471 entries in the book, which is the second largest number among the stocks in our sample. The average cost of *ex-ante* liquidity is reported for five representative percentages of the book for each stock available. Table 1 also contains the average relative bid-ask spread for each of the five percentages, and the average depth.

As expected, the average cost of *ex-ante* liquidity is increasing in the percentage of the book considered. This of course implies that it becomes more expensive to buy and sell simultaneously a higher percentage of the book. Moreover, TEF, the largest stock in the sample is also the most liquid asset independently of the percentage of the book we take. Similarly, TPZ, the smallest stock is also the less liquid asset for all percentage levels. The remaining stocks do not maintain a direct relationship between market value and the cost of liquidity. In any case, their liquidity costs, for a given percentage of the book, are quite similar. It should also be noted that, for all five stocks, the percentage increase in liquidity costs between the 1% and 25% of the book tends to be very similar to the percentage increase between 25% and 100%. This suggests that the increase in *ex-ante* liquidity costs changes very rapidly once trading moves from the first percentage level.

It is interesting to point out that the ranking of liquidity within each percentage level of the book is practically the same we would obtain by just observing the relative bid-ask spread. However, it is important to note that, on average, the relative bid-ask spread systematically overvalues the true cost of liquidity given by our measure. It should be recalled that the relative bid-ask spread is the cost of simultaneously buying and selling one unit of the stock. Given that the true cost
of liquidity is increasing in size, one should expect the relative bid-ask spread to overvalue the true liquidity cost even for the one percentage level reported in our work. This is actually the case.

### Table 1: Average *Ex-Ante* Liquidity Cost, Relative Spread and Depth for Five Percentages of the Book

The *ex-ante* liquidity cost represents the costs of buying and selling simultaneously a given amount of shares. It is given by $L(n) = \frac{AC(n) - AR(n)}{V^*}$, where $AC(n)$ is the average cost, $AR(n)$ is the average revenue, and where $V^*$ approximates the true value of the asset. In the table below, the average of the liquidity cost over all entries in the open limit order book during July 1999, and for each stock available in the sample is calculated for five representative percentages of the book. The relative spread is just $(P_a - P_b) / \frac{[P_a + P_b]}{2}$. Again, it is obtained as the average over the whole sample period for each stock in our cross-sectional sample. Depth is also given as the corresponding average for all trading days during July 1999. Both are calculated within each of the five percentages of the book employed in the table.

<table>
<thead>
<tr>
<th>Book (%)</th>
<th>ACE</th>
<th>ACX</th>
<th>CTG</th>
<th>TPZ</th>
<th>TEF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(12109)</td>
<td>(9603)</td>
<td>(22823)</td>
<td>(30471)</td>
<td>(86937)</td>
</tr>
<tr>
<td>1%</td>
<td>0.00286</td>
<td>0.00266</td>
<td>0.00295</td>
<td>0.00359</td>
<td>0.00074</td>
</tr>
<tr>
<td>25%</td>
<td>0.00471</td>
<td>0.00437</td>
<td>0.00438</td>
<td>0.00726</td>
<td>0.00136</td>
</tr>
<tr>
<td>50%</td>
<td>0.00613</td>
<td>0.00573</td>
<td>0.00562</td>
<td>0.00997</td>
<td>0.00184</td>
</tr>
<tr>
<td>75%</td>
<td>0.00738</td>
<td>0.00698</td>
<td>0.00674</td>
<td>0.01224</td>
<td>0.00207</td>
</tr>
<tr>
<td>100%</td>
<td>0.00855</td>
<td>0.00822</td>
<td>0.00783</td>
<td>0.01436</td>
<td>0.00267</td>
</tr>
</tbody>
</table>

Panel A: Average *ex-ante* liquidity cost

<table>
<thead>
<tr>
<th>Book (%)</th>
<th>ACE</th>
<th>ACX</th>
<th>CTG</th>
<th>TPZ</th>
<th>TEF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(12109)</td>
<td>(9603)</td>
<td>(22823)</td>
<td>(30471)</td>
<td>(86937)</td>
</tr>
<tr>
<td>1%</td>
<td>0.00295</td>
<td>0.00273</td>
<td>0.00302</td>
<td>0.00381</td>
<td>0.00078</td>
</tr>
<tr>
<td>25%</td>
<td>0.00630</td>
<td>0.00589</td>
<td>0.00574</td>
<td>0.01032</td>
<td>0.00190</td>
</tr>
<tr>
<td>50%</td>
<td>0.00871</td>
<td>0.00829</td>
<td>0.00793</td>
<td>0.01476</td>
<td>0.00273</td>
</tr>
<tr>
<td>75%</td>
<td>0.01096</td>
<td>0.01067</td>
<td>0.00999</td>
<td>0.01873</td>
<td>0.00349</td>
</tr>
<tr>
<td>100%</td>
<td>0.01334</td>
<td>0.01342</td>
<td>0.01210</td>
<td>0.02251</td>
<td>0.00434</td>
</tr>
</tbody>
</table>

Panel B: Average relative spread

<table>
<thead>
<tr>
<th>Book (%)</th>
<th>ACE</th>
<th>ACX</th>
<th>CTG</th>
<th>TPZ</th>
<th>TEF</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>347.10</td>
<td>129.24</td>
<td>113.88</td>
<td>1104.76</td>
<td>594.53</td>
</tr>
<tr>
<td>25%</td>
<td>8677.46</td>
<td>3231.05</td>
<td>2847.00</td>
<td>27618.97</td>
<td>14863.30</td>
</tr>
<tr>
<td>50%</td>
<td>17354.93</td>
<td>6462.10</td>
<td>5693.99</td>
<td>55237.95</td>
<td>29726.61</td>
</tr>
<tr>
<td>75%</td>
<td>26032.39</td>
<td>9693.15</td>
<td>8540.99</td>
<td>82856.92</td>
<td>44589.91</td>
</tr>
<tr>
<td>100%</td>
<td>34709.86</td>
<td>12942.19</td>
<td>11387.99</td>
<td>110475.89</td>
<td>59453.21</td>
</tr>
</tbody>
</table>
Generally speaking, average depth provides similar information to the previous two measures of liquidity. However, TPZ presents an unusually large number of shares available that is probably related to changes in the ownership composition experienced by this company in July 1999.

Table 2 reports the average ex-ante cost of buying and selling a given percentage of the book during July 1999 for our five companies. As implied by expression [7], both figures add to the total average cost of liquidity provided by table 1. It might be pointed out that, in this sample, the (liquidity) cost of buying is higher than the (liquidity) cost of selling. This is probably associated with the declining market during July 1999. Of course, market conditions should determine whether is more expensive in terms of liquidity cost to buy or to sell a given percentage of the book.

Initially, we construct the liquidity functions for each stock at every instant in which a modification in the limit order book is observed. It would be not practical however to report all available instants in the book. Hence, we summarize the re-

<table>
<thead>
<tr>
<th>Book (%)</th>
<th>ACE (12109)</th>
<th>ACX (9603)</th>
<th>CTG (22823)</th>
<th>TPZ (30471)</th>
<th>TEF (86937)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>0.00157</td>
<td>0.00163</td>
<td>0.00159</td>
<td>0.00238</td>
<td>0.00036</td>
</tr>
<tr>
<td>25%</td>
<td>0.00250</td>
<td>0.00254</td>
<td>0.00226</td>
<td>0.00417</td>
<td>0.00066</td>
</tr>
<tr>
<td>50%</td>
<td>0.00324</td>
<td>0.00320</td>
<td>0.00293</td>
<td>0.00556</td>
<td>0.00091</td>
</tr>
<tr>
<td>75%</td>
<td>0.00391</td>
<td>0.00376</td>
<td>0.00356</td>
<td>0.00676</td>
<td>0.00113</td>
</tr>
<tr>
<td>100%</td>
<td>0.00456</td>
<td>0.00429</td>
<td>0.00418</td>
<td>0.00787</td>
<td>0.00135</td>
</tr>
</tbody>
</table>

Panel A: Buy ex-ante liquidity cost

<table>
<thead>
<tr>
<th>Book (%)</th>
<th>ACE (12109)</th>
<th>ACX (9603)</th>
<th>CTG (22823)</th>
<th>TPZ (30471)</th>
<th>TEF (86937)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>0.00129</td>
<td>0.00103</td>
<td>0.00137</td>
<td>0.00121</td>
<td>0.00038</td>
</tr>
<tr>
<td>25%</td>
<td>0.00221</td>
<td>0.00183</td>
<td>0.00212</td>
<td>0.00309</td>
<td>0.00070</td>
</tr>
<tr>
<td>50%</td>
<td>0.00289</td>
<td>0.00253</td>
<td>0.00269</td>
<td>0.00441</td>
<td>0.00093</td>
</tr>
<tr>
<td>75%</td>
<td>0.00346</td>
<td>0.00322</td>
<td>0.00318</td>
<td>0.00548</td>
<td>0.00937</td>
</tr>
<tr>
<td>100%</td>
<td>0.00399</td>
<td>0.00393</td>
<td>0.00365</td>
<td>0.00649</td>
<td>0.00133</td>
</tr>
</tbody>
</table>

Panel B: Sell ex-ante liquidity cost

The ex-ante liquidity cost of buying a given number of shares is given by LB(n) = [AC(n) – V*]/V*, where AC(n) is the average cost, and V* approximates the true value of the asset. The liquidity cost of selling a given number of shares is given by LS(n) = [V* – AR(n)]/ V*, where AR(n) is the average revenue. In the table below, the average of the liquidity cost of buying and selling over all entries in the open limit order book during July 1999, and for each stock available in the sample is calculated for five representative percentages of the book.
results by taking the mean of the normalized liquidity function for each quarter of an hour. If there are no modifications in the limit order book during a given quarter, we take the values of the last instant in the previous quarter.

Figure 1 shows our liquidity functions calculated according to expression [3] for each of the five stocks in our sample. As mentioned above, they are constructed as percentage of the book, and they are reported for every fifteen minutes and for five representative percentages of the book. Each point on the graph indicates the ex-ante cost of buying and selling simultaneously a given percentage of the book at a single quarter of an hour during July 1999.

Figure 1 confirms that the ex-ante cost of liquidity is increasing in the number of shares, and that the behavior of the cost does not remain constant either throughout the day, or across the stocks with different market values. In the case of TEF, the largest Spanish company traded in the stock exchange, there seems to be a slight evidence of higher liquidity costs at the beginning of the day, although this is particularly true for high percentages of the book. It is not clear that there exists, generally speaking, the typical intraday U-shaped pattern found by Mclinish and Wood (1992), Lin, Sanger and Booth (1995), Chung, Van Ness and Van Ness (1999), or Pascual, Escribano and Tapia (2004).

For mid-market values companies like CTG and ACE, however, there seems to be a clearer evidence of a U-shaped for percentages higher than 25% of the book. ACX presents a rather clear (reversed) J-shaped pattern, and TPZ seems to have a decreasing cost of liquidity at the end of the day for all percentages of the book.

Finally, figure 2 presents the liquidity function estimated by equation [3] for all five companies at a given quarter of an hour, and for all percentages of the book. The first graph displays the liquidity cost as a function of the percentage of the book at 10:15 (beginning of the day), while the other two contain the liquidity function at 14:00 (middle of the day) and 17:00 (end of the day) respectively.

We again point out that, of course, liquidity functions are increasing in the percentage of the book. At the beginning of the day, and almost independently of the percentage of the book considered, the cost of liquidity is higher the lower the market value of the company. However, and this is also true for the cost of liquidity at the end of the day, there exists some changes in the cost of liquidity across companies relative to its market value. TPZ is not the less liquid company at either 10:15 or 17:00. However, this company is the less liquid asset for the rest of the fifteen minute intervals available.

Between 10:30 and 11:15, and independently of the percentage of the book, the larger the market value of the company the lower the cost of liquidity. Between 11:30 and 16:45, the same result is obtained except for ACX that becomes increasingly more liquid relative to other companies in the sample. This suggests that transactions for this company should become relatively larger than the rest of the firms in our sample. The continuously decreasing cost of ex-ante liquidity should motivate and facilitate more trading among investors. In general, therefore, it is not true that the largest market value, the more liquid the company becomes. In any case, it should be noted that our measure is able to compare without ambiguities the cost of liquidity among companies for a given time of the day and a given percentage of the book or volume. This is a very important result.
Understanding the *ex-ante* cost of liquidity in the limit order book: a note

Figure 1: The Liquidity Function: Summarizing the Information in Five Percentages of the Book
Moreover, we know that the *ex-ante* cost of liquidity as measured by the high of the vertical axis indicates the cost of buying and selling simultaneously a given percentage of the book. This is the concept emphasized in expression [3]. However, it should be noted that our measure is able to incorporate another dimension of liquidity. The slope of the functions reported in figure 2 suggests the variation in volume needed to move the spread differential. At the buy side of the market, the slope indicates how much volume is needed to move the ask prices relative to the true value of the asset. The reasoning would also be valid at the selling side of the market. It is very interesting to note that the empirical results regarding both the slope and the (high) vertical dimension of our measure are consistent. We observe how the difference in the cost of liquidity becomes increasingly higher the higher is the percentage of the book considered. This is a consequence of the highest slope for the less liquid companies.
Figure 2: The Liquidity Function at Three Time Intervals
4. CONCLUSIONS

This paper has proposed and estimated a new measure of liquidity for markets driven by orders. The full availability of the limit order book is needed. However, once these data are observed, a very useful measure of liquidity is easily estimated. This single measure completely characterizes the *ex-ante* cost of liquidity. This is a very important issue, since it avoids the traditional ambiguities confronted by researchers when using either the relative bid-ask spread or depth. Moreover, it aggregates all information available in the limit order book into a single measure that represents the level of committed liquidity in a given asset, and it also facilitates comparisons among stocks and markets. At the same time, as pointed out by Benston, Irvine and Kandel (2000), these types of measures should be useful in all types of markets, although stock exchanges in which high proportions of liquidity are really committed rather than hidden would be the most benefited. This suggests the importance of mechanisms with an electronic open limit order book.

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RESUMEN
El presente trabajo estima una nueva medida de coste de liquidez de los activos financieros en un mercado dirigido por órdenes. Esta medida, denominada función de liquidez, recoge el coste ex-ante de comprar y vender simultáneamente una determinada cantidad de acciones haciendo uso de toda la información ofrecida por el libro de órdenes. De esta manera se superan las dificultades que la consideración por separado de la horquilla de precios o la profundidad ocasiona sobre la caracterización de la liquidez de los diferentes activos.

Palabras clave: función de liquidez, coste de liquidez, libro de órdenes límite, horquilla de precios, profundidad.

Clasificación JEL: G14.