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NONPARAMETRIC ESTIMATION OF INTEREST RATES PROCESSES IN EUROPE*

TERESA CORZO SANTAMARÍA
Universidad de Navarra

JAVIER GÓMEZ BISCARRI
UCLA

Interest rate dynamics are crucial in modern economic and financial research. There has recently been an increased effort in trying to develop models of interest rate behavior. Unfortunately many of these models are based upon very restrictive assumptions: they often hypothesize that the continuous-time process followed by the instantaneous short-term interest rate is linear on its drift and has constant volatility. We study the case of two European Monetary Union participant countries where we model interest rates as converging towards a central, European rate. With nonparametric techniques we show conclusive evidence of nonlinearities in the drift and heteroskedasticity. Our results confirm those of previous nonparametric literature. Due to the special current situation of the EMU block we use as a conditioning variable the spread between the Domestic and the European short-term interest rate. We study the Italian and Spanish cases.

Key words: nonparametrics, empirical, interest rate processes, continuos time.

JEL classification: E43, C14, F36.

Interest rates play a key role in any economy. They are important for a correct domestic and foreign capital markets equilibrium and may therefore greatly influence the efficiency of the allocation of resources in that economy. In addition, the behavior of the term structure of interest rates is one of the topics in financial economics that has attracted more attention during the last decade. This has probably been induced by the importance achieved by the amount of trade in fixed income securities and derivatives.

Since the seminal papers by Merton (1973) and Vasicek (1977), many interest rate models have been developed. In the simplest form, interest rates have

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been modeled as one-factor Markovian processes, although nowadays it is common to find models in the literature where interest rates are modeled as two, three or even four factor processes[1].

While there has been a big effort in developing models and closed form solutions for derivatives or other financial assets based upon the dynamics of the interest rate, we still have little guidance in choosing the right model. Strictly speaking there are some unwarranted restrictive assumptions underlying the models. Almost all of them assume, for example, that interest rates have a linear drift. The models also differ on the assumptions concerning the diffusion. Vasicek (1977), for example, specifies that the instantaneous volatility of the spot rate process is constant, while Cox, Ingersoll and Ross (CIR 1985) assume that the instantaneous variance is a linear function of the level of the spot rate, so the standard deviation is a function of the square root of that level.

These simplifications constitute the price to pay for the use of a convenient and simple formula to price financial assets. Recent empirical research has showed that the drift of the short rate is generally nonlinear[2], the volatility is not constant and it usually depends upon the level of the interest rates, and the densities for the errors are nonnormal. Pagan, Hall and Martin (1996) provide an excellent summary of the stylized facts of interest rates behavior and how most of the existing models fail to account for them. Since both the drift and the diffusion are key for accurate estimation of the future evolution of interest rates and when pricing interest rates derivatives[3], it is therefore of extreme importance to model them correctly.

In response to the problems mentioned, researchers have started to use semiparametric and nonparametric techniques to avoid the inconvenience of the arbitrary restrictions that parametric functional forms impose on the underlying processes. Some examples of the application of these methodologies to financial research are Aït-Sahalia (1996a, b), Stanton (1997) and Boudokh *et al.* (1999).

In this paper we study, through the use of nonparametric techniques, the features of the data used to proxy for the short-term interest rate process followed by a European economy. We understand a European economy to be one participating in the EMU block, following the intuition developed by Corzo and Schwartz (2000) in the convergence model.

This paper's application focuses on the estimation of the processes followed by the Italian and Spanish short-term interest rates as concrete examples for countries that experienced the interest rate convergence. It is not the purpose of this study to discuss the wide range of applications for the convergence model, but rather to study more thoroughly the characteristics of the underlying interest rates data set.

(1) See, i.e. Longstaff and Schwartz (1992) for a two factor model, and Balduzzi *et al.* (1998) and Chacko (1997) for three and four factor ones.

(2) See, for exemple, Aït-Sahelia (1996b) and Pfann *et al.* (1996).

(3) If the underlying state variable were itself an asset price we know that drift would not matter (it does not intervene in the pricing of the derivative), but misspecifying the drift may lead to inconsistent estimation of the diffusion.

Discrete data are used and so the estimation still does rely in replacing the continuous-time diffusion processes by some discrete approximation.

That approximation is equivalent to the first order term in the stochastic Taylor Series expansion that Stanton (1997) used to estimate continuous time interest rate processes. He presents simulation evidence that with weekly data, this first order approximation is accurate enough. In fact, we tried adding the second order term in our estimations and the results were almost indistinguishable. The advantage of using Stanton's method, though, is that we can derive the order of the errors, and we know that by expanding the stochastic TSE we can make that order arbitrarily small. We find supportive evidence of nonlinear drift and heteroskedastic diffusion, which makes parametric (linear) models, estimated by OLS or GMM, inaccurate. We also find that, once these characteristics of the data have been taken care of with nonparametric techniques, the normality of residuals increases.

The interest rate processes are estimated, following the convergence model, as functions of the spread between the domestic and the central short-term rate, then just as functions of the short-term domestic rate. In both cases the behavior of the interest rates shows nonlinearities, hoteroskedasticity and non-normal density functions for the residuals. However, in the Italian case the use of the spread as explanatory variable helps markedly in explaining changes in the interest rate. Furthermore, the density function of the residuals displays less variance; in the Spanish case the same evidence appears although not as strong.

The paper is organized as follows: in section 1 we present the model to be estimated and review the convergence model and include a subsection with the estimation methods that will be used in the empirical analysis. In section 2 we present the data and the results of the estimation. Finally in section 3 we conclude.

1. The model

Interest Rates in European countries participant in European Monetary Union (EMU) move no longer in isolation but rather move together as a consequence of having fixed bilateral exchange rate against the Euro in January 1999. Movements in European interest rates have repercussions in domestic interest rates and by extension in interest rate derivatives. Our aim is to take into account this influence by assuming that the change in the level of the short-term interest rate of countries participating in EMU can be explained by the spread between the domestic and the short-term central rate.

With fixed exchange rates, risk-free interest rates across countries should be the same. Even before the formation of the EMU block we could observe how interest rates in the participating European countries were converging. We understand here the risk-free interest rate as being the interest rate yielded by the Treasury debt. The State is supposed to be the most reliable institution with hardly any chance of default, but *stricto sensu* different countries have different risks just as different companies in the same country. So, even with a single currency interest rates may be slightly different reflecting different sovereign and credit risks. This difference may become specially notorious for medium and long term debt.

So far short-term interest rates have been the key element in every term-structure dynamics model. For this reason we focus on its evolution, although we could study in a similar way the medium and long term rate. Relating the changes of the short-term domestic rates to its spread with the European short-term rate makes sense given the actual political and economic conditions. Because participant countries will be allowed to issue domestic debt contemporaneously with the debt issued by the European Central Bank, it is very likely that small differences may still exist even when the single currency is implemented.

Some caveats are in order. Besides the differences among the EMU Central Banks official interest rates –as we have already pointed out due to the different risks and terms– there are a number of countries that will not join the union now, and whose interest rates are still far away from the European, central rate. Some of these countries (i.e. United Kingdom) have already manifested their will to be in EMU in a near future. Our model is specially suitable for them, since they will have to follow a convergence process on the interest rates. With the EMU in place, the Euro-11 countries are a close match for the United States in economic and financial terms. It will be interesting to study how the interest rates of the two currencies move together and how the spread behaves. Again, the philosophy of our model can be used[4].

The process we are going to estimate here is a one-factor model. We use the difference (spread) between the domestic and the central short-term interest rates. We believe that, given the convergence requirements and the evolution of the rates, shown in figure 1, this specification captures in a convenient way the process that interest rates have followed. The following specification corresponds both to a model where the domestic interest rate is a stochastic mean reverting process, a terminology more in line with financial research, and to a model where the European and domestic interest rates are cointegrated with cointegrating vector (1, -1) and a common drift, the European rate, more in the time series econometrics context. In both cases, the influence of the error correction term is not assumed to be necessarily linear.

We represent r_d as a continuous-time diffusion process, satisfying a timehomogeneous SDE,

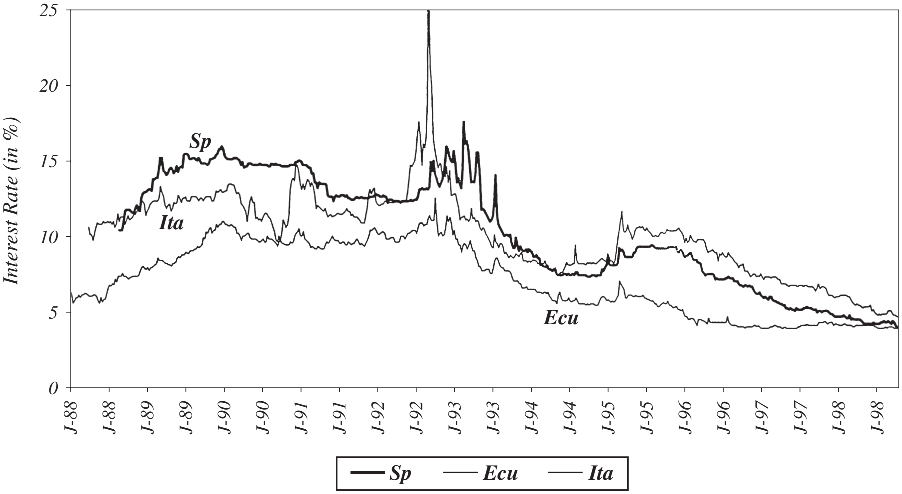
$$dr_d = m\,(r_e - r_d)\,dt + \sigma\,(r_e - r_d)\,dz_d \qquad [1]$$

where r_d is the domestic rate, r_e the European rate and dz_d is an increment to a Wiener process.

We provide a model-free estimation for [1] using the nonparametric approach to avoid having to specify functional forms for m and σ. Previous work in this area is Stanton (1997) or Broadie *et al.* (1996).

(4) It is not clear yet how the dollar's dominant role in the world financial system adjusts with the Euro being the world's second biggest currency. In this sense we do not know which of the two currencies interest rate will be the leading, or if they will just influence each other in search of an equilibrium.

Figure 1: SHORT-TERM INTEREST RATE EVOLUTION



Following the usual practice in these type of studies, we estimate the continuous-time model using a discrete-time version of the process that corresponds to the first order term in its stochastic Taylor series expansion [Stanton (1997)]:

$$\Delta r_{d,t} = r_{d,t+\Delta t} - r_{d,t} = m[(r_e - r_d)_t]\,\Delta t + \sigma\,[(r_e - r_d)_t]\sqrt{\Delta t}\;\xi_{d,t} + o\,(\Delta t) \qquad [2]$$

where $\xi_{d,t}$ is a standard normal variable. The $r_{d,t}$ are observations of the Italian and Spanish short term interest rates at each moment of time, and $(r_e - r_d)_t$ are the values of the spread between the domestic –Spanish or Italian– and the European short term interest rates at each moment in time.

We estimate

$$\Delta r_{d,t} = m\,[(r_e - r_d)_t]\,\Delta t + u_t \qquad [3]$$

where $u_t = r_{d,t+\Delta t} - r_{d,t} - m[(r_e - r_d)_t]\,\Delta t$ can be shown to be errors of $O(\Delta t)$. As we mentioned before, the order of these errors could be arbitrarily reduced by including higher order terms of the stochastic TSE, but this does not seem to lead to substantial gains in the empirical applications.

We obtain estimates of the conditional expectation of the change in the interest rate

$$\hat{E}\,[\Delta r_{d,t} \mid (r_e - r_d)_t] = \hat{m}\,(r_e - r_d)_t \qquad [4]$$

and of the conditional variance

$$\hat{\sigma}^2_{\Delta r_{d,t}} = v\hat{a}r\,[u_t \mid (r_e - r_d)_t] \qquad [5]$$

Later in the paper we compare those results with those obtained using the specifications $\hat{E}\,[\Delta r_{d,t} \mid r_{d,t}] = \hat{m}\,(r_{d,t})$ and $\hat{\sigma}^2_{\Delta r_{d,t}} = v\hat{a}r\,[u_t \mid r_{d,t}]$.

2. Estimation

We proceed to estimate nonparametrically the process [1] presented in section 1. As concrete examples for the domestic economy we take the Italian and Spanish cases and we replace r_d for r_i or r_s[5]. For the Italian case, i.e., we have therefore $dr_i = m(r_e - r_i) + u$, and we estimate $E[dr_i \mid r_e - r_i] = \hat{m}\,(r_e - r_i)$ and $var\,[u_t \mid r_e - r_i] = E[u^2 \mid r_e - r_i] = \hat{\sigma}^2(r_e - r_i)$.

Technical details can be found in Scott (1992), Bosq (1998) and the excelent textbook by Pagan and Ullah (1999). For all estimations of density functions we will use a unidimensional Rosenblatt-Parzen kernel estimator. In the case of estunation of moments of a variable y conditional on another variable x we will use the Nadaraya-Watson estimator[6].

Second moments can be estimated after the nonparametric estimation of $m(x)$. If the residuals from the nonparametric regression are

$$\hat{u}_i = y_i - \hat{m}\,(x_1) \qquad [6]$$

then, since $\sigma^2\,(u \mid x) = E^2\,[u^2 \mid x] - (E\,[u \mid x])^2$ and we assumed $E\,[u \mid x] = 0$ the conditional variance is just the conditional expectation of the squared residuals and thus can also be estimated using a Nadaraya-Watson type estimator.

Our choice has been to use a Gaussian kernel for all unidimensional estimations since the election of the kernel weighting function has been shown to be of relatively little importance [Scott (1992)]. The choice of the smoothing parameter *h is* more relevant: If the function is oversmoothed or undersmoothed (that is, if h is higher or smaller than some optimal value) the actual estimation might differ substantially from the true function $f\,(z)$. A practical rule that works optirnally in most situations is *Silverman's rule of thumb* $h_i = 1.06 \cdot \hat{\sigma}_i \cdot N^{\frac{-1}{d+4}}$ that is very close to the optimal one when using a Gaussian kernel. This window width will be used for calculations throughout the paper[7].

(5) These two economies belong to EMU nowadays and experienced interest rate convergence during the sample period, so they are adequate candidates for our model.

(6) Results using the Local Linear Regression estimator are available upon request. These results do not differ substantially, so we decided to keep the simpler, more well-known Nadaraya-Watson estimator.

(7) We tried cross validation window widths and different constants in the expression for Silverman's rule of thumb. Results did not change significantly.

2.1. Data

The short-term interest rates used are the Italian interbank one-month middle rate, the one-month inter-bank middle rate for the Spanish case, and the middle interest rate for one-month deposits in ECUs.

The Italian series starts on April 8, 1988 and the Spanish one on September 2, 1988. These dates have been determined by the data availability on Datastrearn. We use weekly data. The number of observations are respectively 550 and 529.

Table 1 provides the descriptive statistics for the interest rate data, and figure 1 shows its time evolution. In figures 2, and 3 we have plotted the weekly changes

Table 1: DESCRIPTIVE STATISTICS. WEEKLY DATA

	Mean	Std. Dev.	Skewness	Kurtosis	Max.	Min.
r_e	0.073316	0.024727	0.018713	1.469961	0.126875	0.04
$d_{r.l}$	-0.000112	0.005916	3.050425	110.5801	0.086	-0.0644
r_l	0.103043	0.02761	0.531363	5.67927	0.2769	0.047825
r_e-r_l	0.03008	0.01465	-1.777679	14.69866	0.001875	-0.161587
$d_{r.s}$	-0.000121	0.003438	0.106569	31.53948	0.0307	-0.0272
r_s	0.104749	0.036919	-0.138209	1.655474	0.1775	0.0415
r_e-r_s	0.031433	0.016412	-0.166418	2.412716	0.000875	-0.080625

Figure 2: WEEKLY CHANGES IN THE ITALIAN SHORT RATE PLOTTED AGAINST SHORT RATE ON PRECEDING WEEK

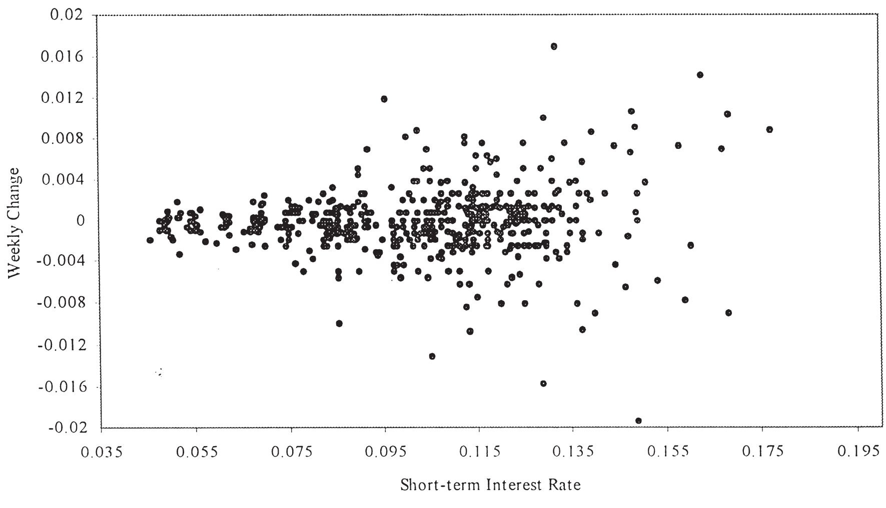
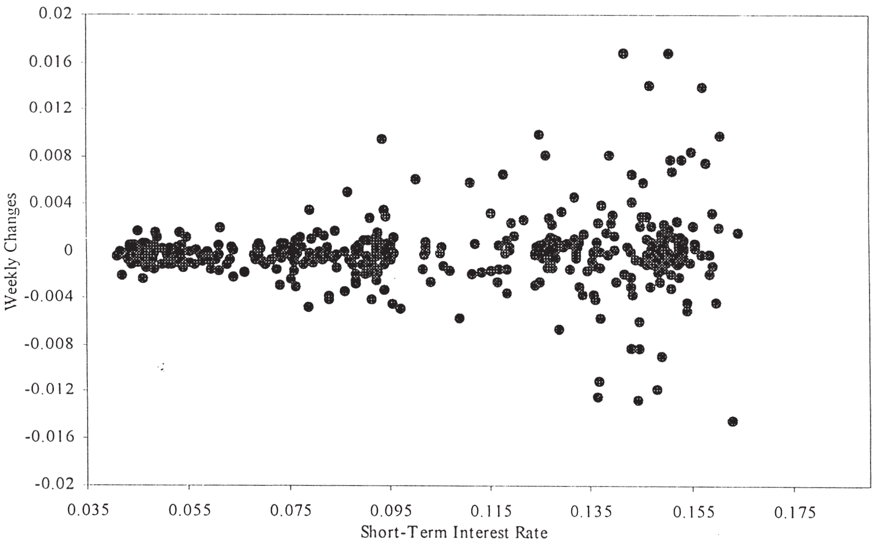


Figure 3: WEEKLY CHANGES IN THE SPANISH SHORT-TERM INTEREST RATE AGAINST SHORT-TERM RATE ON PRECEDING WEEK



in the one-month interest rates of Italy and Spain against the rate on the previous week. As we can see, there is considerable evidence of heteroskedasticity, with the range of the changes increasing as the level of interest rates increases. We will see that our estimation procedure reveals the underlying relationship at the same time that it confirms the above mentioned heteroskedasticity.

2.2. Results

Our results support those of previous research [i.e. Aït- Sahalia (1996a), Stanton (1997)]. For the two cases under study and conditioning on the spread of the short-term interest rate with the European rate we find nonlinear drifts and heteroskedasticity in the diffusions. The density functions of the conditional residuals are nonnormal.

The drifts estimated nonparametrically, $E(dr_d \mid r_e - r_d) = \mu(r_e - r_d)$ for $d = i, s$ are displayed in figures 4 and 5. We also plot an OLS estimation to provide some intuition of the approximations errors committed when parameterizing the processes. The nonlinear behavior is evident from the graphical representation. This evidence contradicts the classical assumptions of linear behavior of interest rates processes on their drift, as in Vasicek (1977), CIR (1985) and others. The heteroskedastic behavior of the diffusion terms can be seen in figures 6 and 7. The variance grows when so does the state variable $r_e - r_d$ (in these cases as the difference becomes bigger, in absolute value).

Figure 4: ESTIMATED DRIFT FOR THE SPANISH RATE

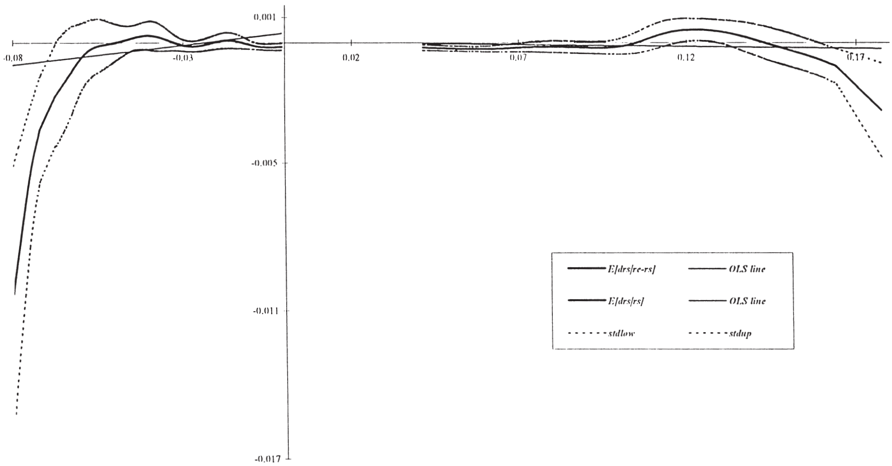


Figure 5: ESTIMATED DRIFT FOR THE ITALIAN RATE

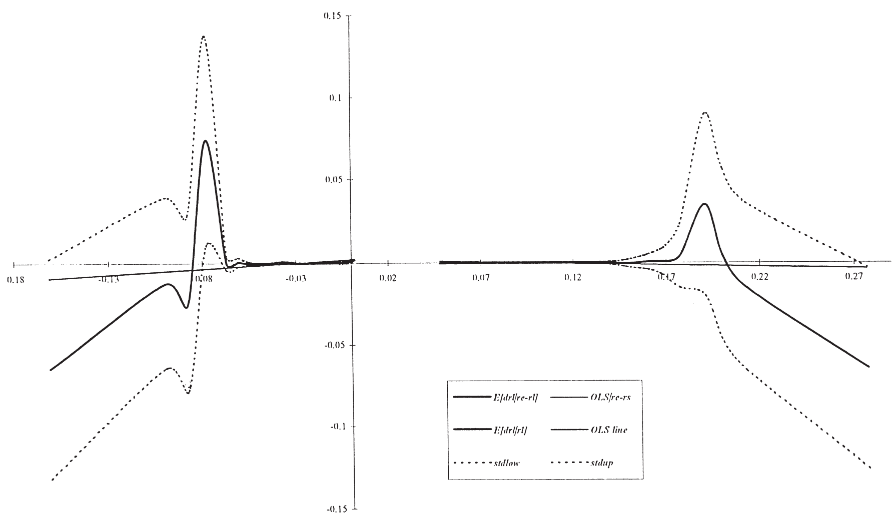
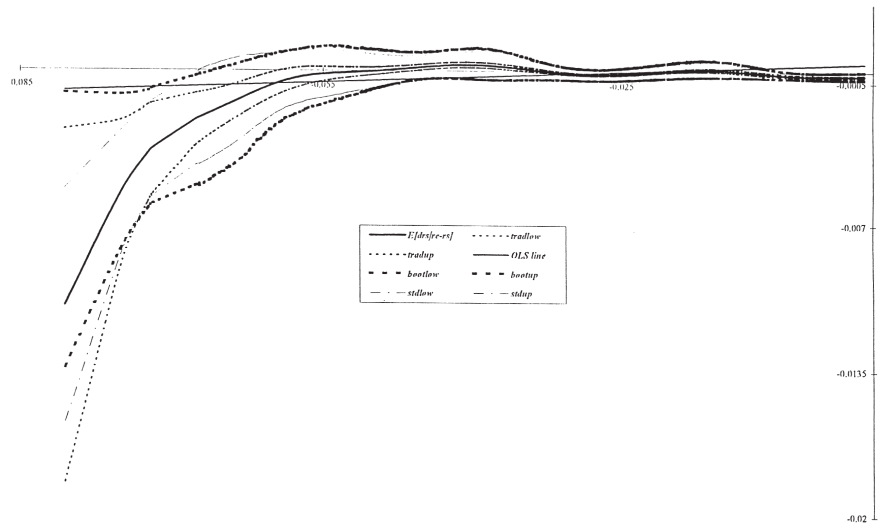


Figure 5a: Comparison of confidence bands (spanish drift)



The previous figures also show the confidence intervals around the conditional nonparametric estimation. We report in these figures the bootstrap standard deviation intervals using 10,000 iterations of the algorithm developed by Künsch (1989). We have included also the bootstrap percentile intervals (using 10,000 iterations of the Künsch algorithm) and the traditional intervals for the Spanish case in figures 5a and 7a as an example to show its behavior. For clarity of exposition we do not include these bands for the rest of the figures. The bootstrap percentile bands are normally above and below the bootstrap standard deviation bands, yielding wider intervals and the traditional bands tend to be too close to the estimated function, as they do not account for the possible dependencies in the data. The significance level is 5%. For more details on the bootstrap bands [see Efron and Tibshirani (1993)].

Results regarding the drift and diffusion suggest that the hypotheses underlying many parametric term structure characterizations are quite inaccurate. And in a wide sense this evidence gives additional support to the usefulness of nonparametric econometrics in the financial economic literature. For example; a role of nonparametric estimation may be to give a guidance to find more correct functional forms for both the drift and the diffusion[8]. In general, nonparametric esti-

(8) For example, in the Spanish case we may think, by looking at the estimation results, that the actual drift is indeed linear up to some level $\overline{r_e - r_s}$ and from that moment on it displays no actual convergence behavior in terms of the spread. A possible explanation for this model would be to interpret $\overline{r_e - r_s}$, as the difference that the markets "think" that should prevail between the Spanish rate and the Euro rate.

Figure 6: ESTIMATED VARIANCE FOR THE SPANISH RATE

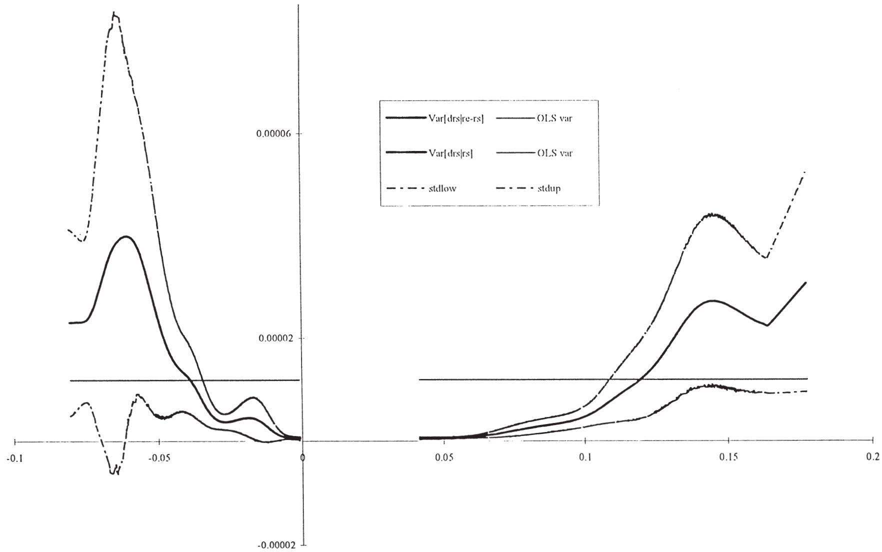


Figure 7: ESTIMATED VARIANCE FOR THE ITALIAN RATE

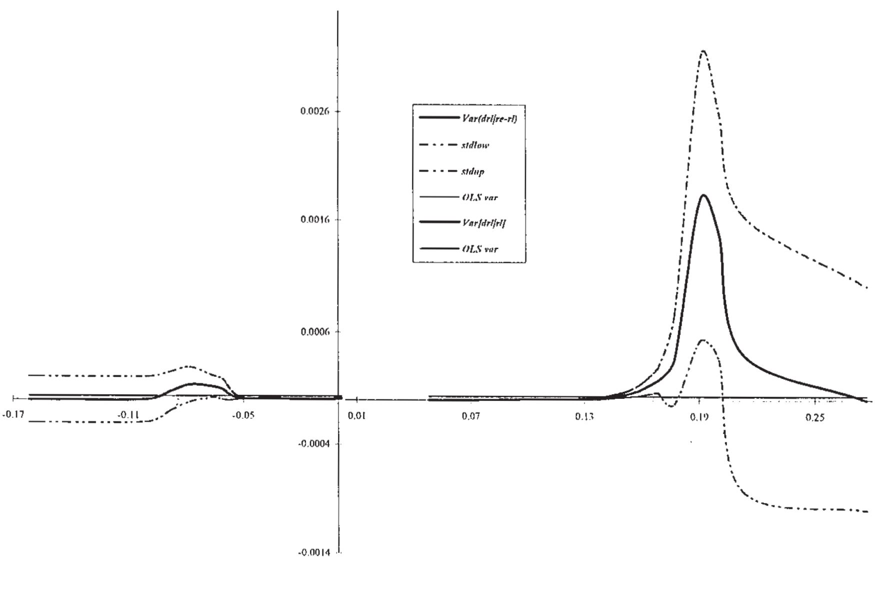
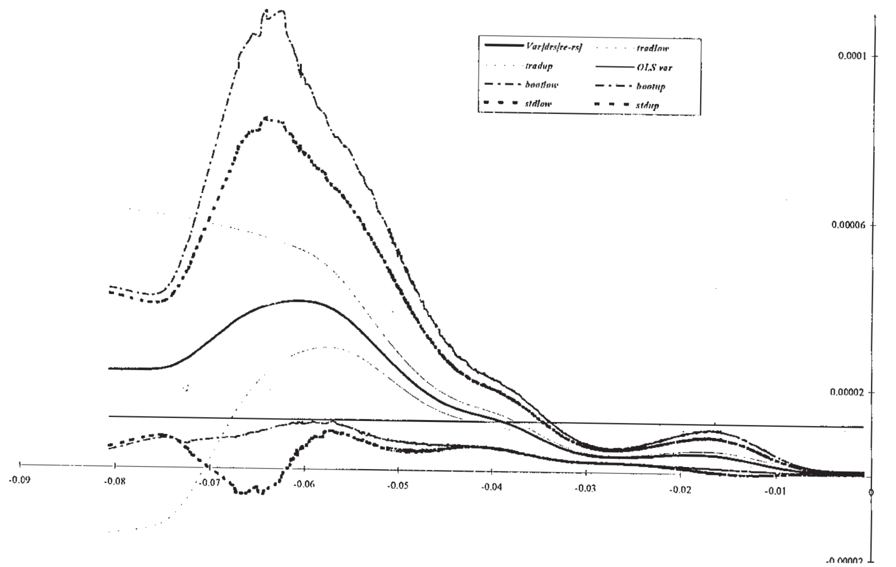


Figure 7a: COMPARISON OF CONFIDENCE BANDS (SPANISH VARIANCE)



mation could be used as a check for previously specified functional forms or as a first step in developing the model for a financial time series.

An additional advantage of the nonparametric technique relies on the diminished effect of outliers on the estimation of other points. The impact of the outliers on the estimation gets weakened as the distance between the outlier and the point gets larger. Or, in other words, precisely because one point is too far from the rest of the sample the procedure will treat it as an outlier, and assign a very small weight to that observation. For instance, lets look closely at the Italian drift. Four outliers account for the ups and downs of estimation of the drift. Once we have eliminated them, using Robinson's (1988) procedure, there is no clear convergence process remaining. The OLS slope is significantly different from zero due to the impact of the outliers, but with the nonparametric estimation we can see that more accurate results, now unaffected by the outliers, show the inexistence of a clear process of convergence.

As an additional exercise we did the same nonparametric estimations conditioning only on the level of the domestic short-term interest rate, instead of using the spread. The results of the drifts and diffusions in the two cases still lead us to the same conclusions of nonlinearities and heteroskedasticity and can be examined in the same figures as the previous ones.

Figure 8: COMPARISON OF ESTIMATED DENSITIES FOR RESIDUALS (SPAIN)

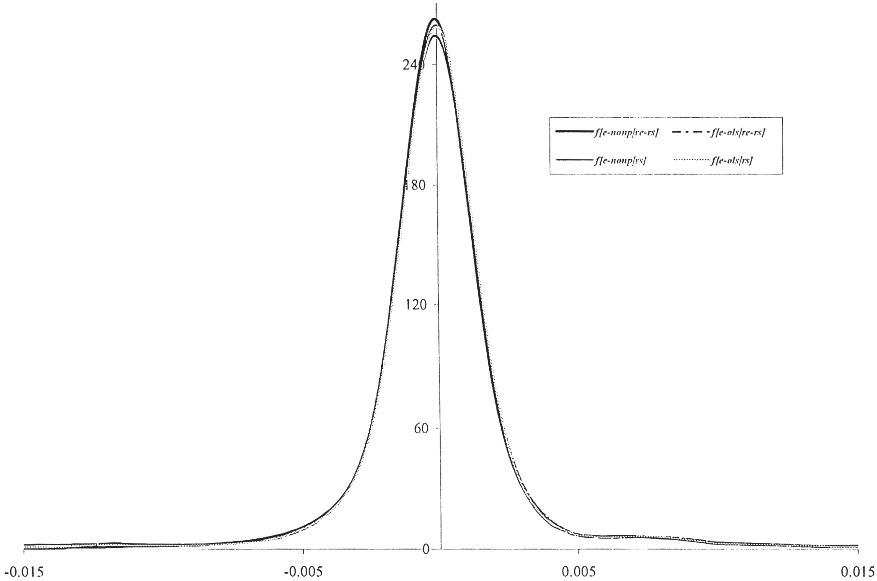


Figure 9: COMPARISON OF ESTIMATED DENSITIES FOR RESIDUALS (ITALY)

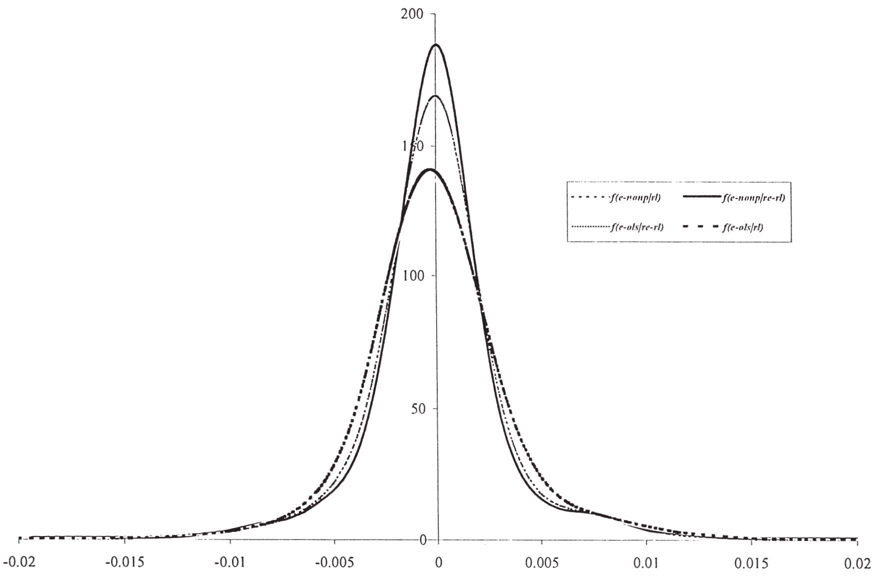
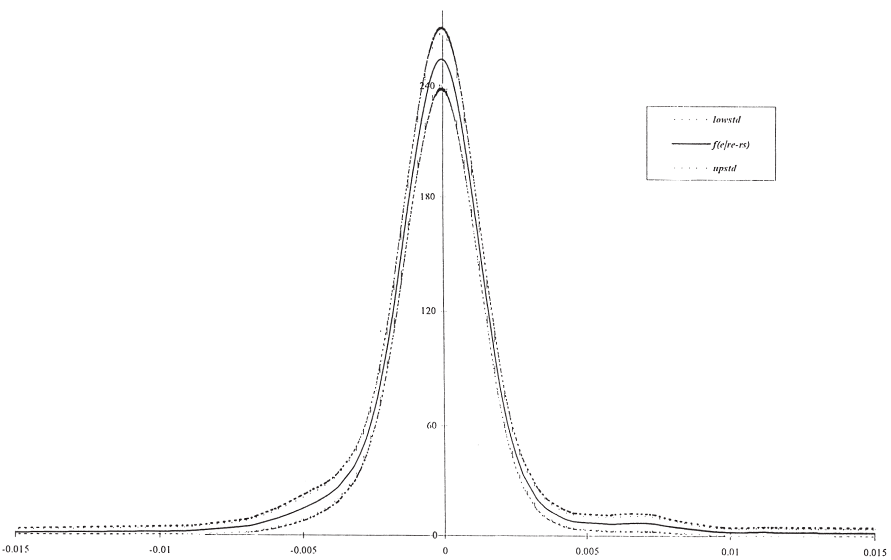


Figure 9a: CONFIDENCE BANDS FOR ESTIMATED DENSITY OF SPANISH RESIDUALS



The estimated density functions of the residuals are showed in figures 8 and 9. Also we examine the residuals after an OLS est imation. Figure 9a includes the bootstrap confidence bands for the Spanish case, these bands, as with the drift and diffusion have been calculated using 10,000 iterations of Künsch's algorithm.

In the Italian case the variance is considerably reduced by the use of the spread in the estimation. This evidence clearly favors our model. Spain shows the same result although not as strong.

We perform a normality test on the residuals in evy case. We test separately the skewness and kurtosis using the asymptotic distribution of the estimator and we test them jointly using the Jarque-Bera test[9]. Results are on table 2. At a 95% confidence level, none of the residuals series seem to fulfill all the conditions to be considered normal. The J.B. test leads us to reject normality in all cases. The Spanish results when conditioning on r_s don't have skewness but they do show evidence of very high excess kurtosis. It is worth noting that the nonnormal behavior of the Italian series is much reduced when using the nonparametric estimation on the spread.

(9) We use at this point Whang's (1998) modification of the Jarque-Bera test for non-parametric residuals.

Figure 10: Comparison of different Kernels

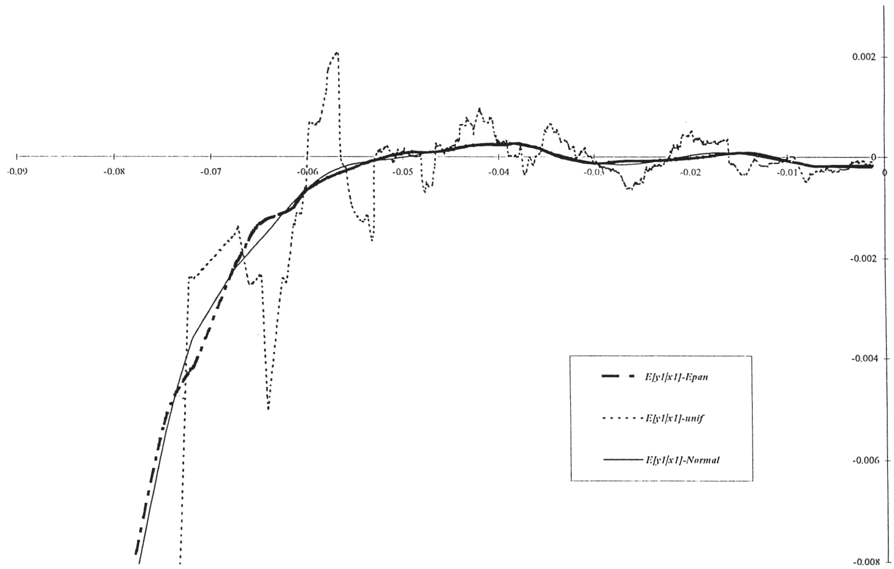


Figure 11: Effect of the choice of the Bandwidth

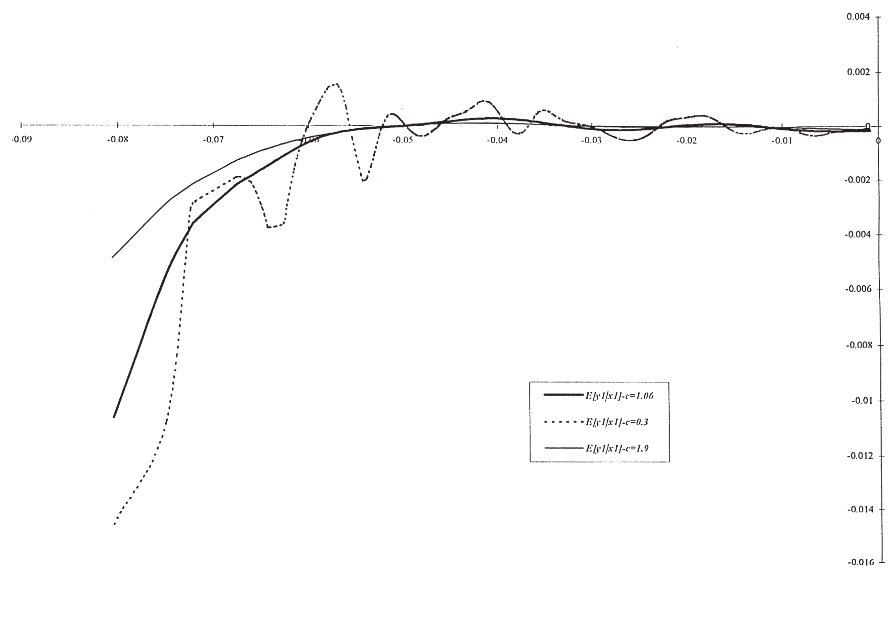


Table 2: RESULTS OF NORMALITY TESTS FOR THE RESIDUALS

	Skewness	Kurtosis	Jarque-Bera
Italy. nonpar. resid. $r_e - r_i$	0.489	8.998	816.59
Italy. nonpar. resid. r_i	1.981	46.27	43269.59
Italy. ols resid. $r_e - r_i$	4.988	116.43	296052.7
Italy. ols resid. r_i	3.92	111.79	271643.6
Spain. nonpar resid. $r_e - r_s$	0.429	31.01	17310.9
Spain. nonpar resid. r_s	0.14	30.16	16264.5
Spain. ols resid. $r_e - r_s$	0.34	28.67	14485.2
Spain. ols resid. r_s	0.15	28.82	14641.4

We have used the asymptotic distribution of the skewness and kurtosis estimators to test normality in each case and Whang's version of the Jarque-Bera test for a joint test of normality.

To the right: cases when we can not reject the normality assumption at a confidence level of 95%.

Finally in figures 10 and 11 we show how the choice of the kernel and window width which could affect estimations, in this case for the drift of the Spanish case: choosing smaller/larger bandwidths leads to very different results, while the choice of kernel (Epanechnikov against a normal) seems to be relatively less important. The uniform kernel, which is a form of histogram estimation, does lead to different results[10].

The results of the estimation allow us to make predictions about the future evolution of the domestic short term interest rate. In a moment of time, given the spread between the domestic and the European rate, we can obtain the drift and diffusion corresponding to that spread –using the results from the estimation of the model–, and then we can calculate the corresponding movement for r_d. The procedure is parallel to forecasts made with parametric models. Forecasting with nonparametric models is not devoid of disadvantages, though. From the practical point of view, they are usually poor out of sample predictors. This comes from the fact that some out of sample observations may have values of the conditioning variable that are not in the range of those used for the estimation. However, this criticism might also apply to parametric models. When one fits a parametric model, especially given the late interest for nonlinear models, the assumption is explicitly being made that the functional relationship effectively applies to values of the conditioning variables outside the sample values. This may not necessarily be true, and in fact one by definition can have no evidence with regards to that

(10) This last kernel gives the same weight to those observations that are contained in a neighborhood (equal to the support of the kernel) around the specific point (e.g. in the case of density estimation it would be counting the number of observations in that "bin" and dividing it by the total number of observations in the sample: therefore the name "histogram estimation").

fact. However, one can use the results of the nonparametric estimation in the same way as one uses a descriptive statistic. They definitely provide clear guidance for the posterior development and fitting of the adequate parametric model [see for example Pfann, Schottman and Tschernig (1996)].

As we just said, for prediction purposes it should be noted that when we want to do some estimation / prediction for values of the conditioning variable far from those in the sample, the prediction could be inaccurate (the estimated function would become flat on those areas, due to the lack of observations in that range within the-sample[11]). We cannot conceal the relevance of this feature of nonparametric estimators in the actual case of the convergence in Europe because interest rates are still falling. In our two cases they are reaching levels below the sample levels, although the sample spreads reflect already almost all the possible situations including minimum spreads.

3. Conclusions

Interest rate processes have been widely studied in the financial economics literature. In this work we use nonparametric techniques for the estimation of the short-term interest rate processes in some European countries and show results that contradict the classical parametric assumptions about interest rates made in most of the preceding literature. We also point out several reasons and evidence that back up the use of nonparametric techniques in financial modelling. Not the least important is the conclusive evidence of nonlinearities in the two drifts and heteroskedasticity in the diflusions. With a 95% confidence level we can reject the normality assumption for the residuals.

The nonparametric estimation for the interest rate processes have been done conditioning on one factor, the spread between the domestic short-term interest rate and the European, central short-term interest rate. We also compare the results with those obtained nonparametrically conditioning on the level of the domestic interest rate itself, and those of an OLS estimation on both the spread and the level. The density functions of the residuals are sensible to the different technique used, displaying less variance with the nonparametric estimation, especially for the Italian case.

An extension of this study focuses on the pricing of interest rate derivatives with this model. It will also be of interest to study the case of the countries belonging to EU but joining EMU in some future date. In these cases the spread may become a revealing variable. Also we note that the relationship between the interest rates on the Dollar and the Euro can be studied using our model.

(11) We can see this phenomenon happening in the Italian case for the bootstrap bands. Due to the low density of observations in the high values of the conditioning variable, most of the bootstrap subsamples do not contain any observations in that range. Therefore, many of the bootstrap estimates in those areas would be estimating a flat function, and when taking the $(\alpha/2)$ percentile we would be getting points from that flat portion.

Several interesting questions arise now from the nonlinearities discovered in the data. As we said before, a first look at the results of the nonparametric estimation may help us reconsider our models to correctly account for the observed behavior.

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RESUMEN

El seguimiento de la dinámica de los tipos de interés a corto plazo es un factor esencial en el estudio de una cualquier economía, y se ha convertido en una herramienta esencial en la construcción de un gran número de modelos en economía financiera. Subyace a muchos de estos modelos la hipótesis de que el proceso seguido por los tipos de interés sigue una tendencia lineal y tiene una volatilidad constante. Usando técnicas noparamétricas y para dos países europeos, España e Italia, mostramos que estas hipótesis son erróneas. La evidencia muestra que los tipos de interés siguen un proceso heteroscedástico con una deriva no lineal. Nuestros resultados coinciden con los de otros estudios noparamétricos. Además dada la situación especial de los países europeos pertenecientes a la Unión Monetaria usamos como variable condicionante el diferencial entre los tipos de interés a corto plazo doméstico y central; comparamos estos resultados con los obtenidos usando sólo el nivel de los tipos domésticos.

Palabras clave: estimación no paramétrica, tipos de interés, convergencia, UME.

Clasificación JEL: E43, C14, F36.